Some Measurements of Spatial Instability Waves in a Round Jet

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This paper discusses some results on the development of the axisymmetric mode of spatial instability waves in a subsonic round jet. The experimental setup is quite similar to that of Crow and Champagne,[3] but the periodic surging imposed on the steady jet flow is kept very weak. Measurements of the amplitude and phase of the velocity fluctuations due to the instability wave across the jet and along the axis are obtained using hot-wire anemometers. The hot-wire signals are phase-averaged by using a waveform eductor, and the magnitude and phase read out from a two phase/vector lock-in amplifier. The experiments are performed at a jet Mach number $M=0.2$, and a Reynolds number $Re=3.317\times10^5$ based on a jet diameter of 2.9 in. Two Strouhal numbers, $St=0.38$ and 1.1, are chosen, and the rms value of the velocity fluctuations at the jet exit due to forcing is kept constant throughout the experiments at 0.04% of the mean flow exit velocity. It is found that the turbulent intensities along the jet centerline are not significantly modified for weak forcing. The growth rate of the instability waves does not follow an exponential form as predicted from linear theory, and the waveform across the jet depends strongly on the mean flow velocity profile. Downstream of the region where the wave amplitude reaches a maximum, a 180° phase reversal across the shear layer is detected indicating the formation of vortices.

Introduction

THERE is sufficient evidence to indicate that the dominant part of the jet noise is generated by the large-scale coherent wave structure of the jet. Over the years there have been a number of experimental studies[1-4] on the development of the wave-like structure in the jet flow, and the growth of instability waves on the jet boundary.[5,6]

The noise generated by the instability waves has been investigated by Tam[7] and Liu[8] for supersonic and subsonic jets. These models use a linear analysis for the growth of spatial instability waves in the jet. It is informative to study the development of these waves experimentally in order to provide a better physical understanding of the orderly structure of the wave-like eddies, and the role they play in noise production.

The experimental setup is quite similar to that of Crow and Champagne.[3] The major differences are in the measuring technique and the amount of forcing at the jet exit. The forcing is kept low in the experiments (0.04% of the mean flow exit velocity) such that the linear stability theory may be expected to be valid in the region where measurements are taken. By using a different measuring technique, it is possible to obtain a Strouhal number preference for maximum amplitude growth of the instability wave. This is because filtered hotwire signals as used by Crow and Champagne[3] cannot detect the instability wave for very weak forcing. Due to the weak forcing, the instability wave is completely buried in the background turbulence. However, using the phase-averaging technique described by Hussain and Reynolds,[9] it is possible to retrieve these waves within the turbulent shear flow from the hot-wire signals.

Two Strouhal numbers are used in this study. Results for the preferred model at $St=0.38$ are given, and some comparisons are made with those at $St=1.1$ The value of $St=1.1$ is chosen so that it is about 3 times larger than the preferred Strouhal number of 0.38, and is slightly less than twice the largest St reported by Crow and Champagne.[3] The present study follows the spatial development of the instability wave by measuring the amplitude and phase along the jet axis, and across the jet at various distances downstream from the nozzle. This was unlike Crow and Champagne, [3] who studied in great detail the effect of St, and the amount of forcing on the amplitude of the instability wave. The results will be useful in the study of large-scale wave-like eddies in jets, and the formulation of a model of aerodynamic sound generation as proposed by Liu.[8]

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Apparatus

In Fig. 1 a schematic of the jet used in the experiments is shown. Forcing at the jet exit plane can be controlled by a 15 in. loudspeaker mounted on the side of the settling chamber. The hot-wire probe is mounted on a 3-axis traversing mechanism. The vertical drive consists of a DISA 55H01 traversing unit with speed controlled by a DISA 32B01 sweep drive unit. The axial and horizontal traverses are motor driven at a fixed speed of 0.05 in/sec. A more complete description of the jet and instrumentations is given in Ref. 10.

Figure 2 shows the setup for turbulence measurements and velocity fluctuations due to the instability wave. The amplitude and frequency of the surging at the jet exit are controlled by a loudspeaker. A sine wave from a signal generator drives the loudspeaker, and at the same time provides a noise-free reference to the waveform eductor (PAR TDH-9) and the two phase/lock-in amplifier (PAR Model 129A). The hot-wire signal is linearized and fed into a signal conditioner which is essentially a filter. The signal is then phase-averaged by the waveform eductor. Corresponding to each sweep of the eductor, a sample of the input signal over 100 equal time intervals is stored in the memory. The hot-wire output gives the 'u' velocity fluctuations due to turbulence and the instability wave. By taking a large number of samples and phase-averaging them, the repetitive signal from the instability wave can be recovered since the phase-average of the turbulent part of the signal is zero.

In some of the measurements in the fully developed region of the jet, an ensemble size of 10×10^3 to 50×10^3 is necessary. The educted signal may still contain noise, and further noise rejection may be obtained by processing the signal through a two phase/vector lock-in amplifier. This allows simultaneous measurements of the magnitude and phase of the educted signal. If the signal is very weak or if the measurements are made in the strongly turbulent region of the jet, it is found that the readings from the lock-in amplifier fluctuate even though a large number of samples are taken in the phase-averaging process. To obtain an average reading, the output is integrated and, depending on the ensemble size used in educting the waveform, an integration time of up to 100 sec is sometimes used.

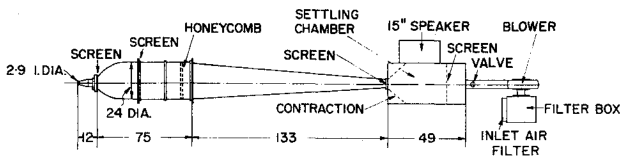

Fig. 1 Schematic of jet (dimensions in inches).

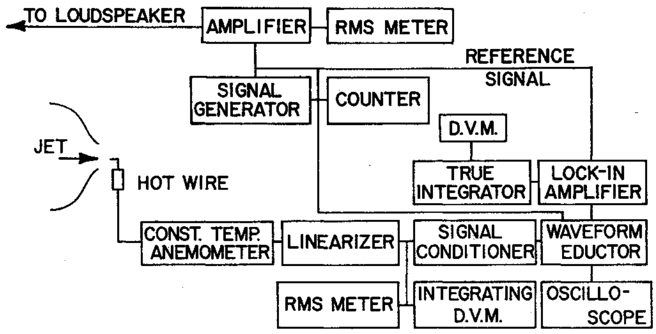

Fig. 2 Block diagram for turbulence and waveform measurements.

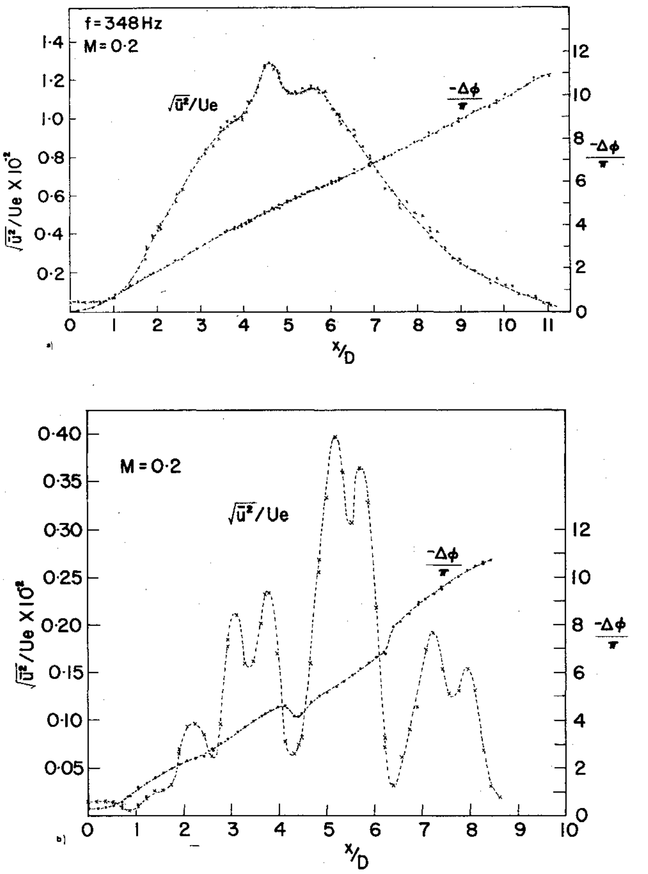

Fig. 3 Magnitude and phase distribution of instability wave along jet axis at $St = 0.38$, a) fundamental mode, b) first harmonic.

Results and Discussions

Throughout the experiments, the jet Mach number M is kept constant at 0.2, and the corresponding Reynolds number is 3.317×10^5. Two forcing frequencies at 348 Hz and 1 kHz are chosen, and these correspond to Strouhal numbers of 0.38 and 1.1, respectively. The rms value of the velocity fluctuations at the jet exit due to forcing is fixed for these two frequencies at a value of 0.04% of the mean exit velocity U_e.

The excellent spark shadowgraphs taken by Brown and Roshko[4] of the instability waves in a plane shear layer show a pronounced large-scale wave structure which originates from the beginning of the mixing layer. The spatially growing waves amplify as they propagate downstream. As the amplitude grows, the vortex layer rolls up to form vortices which finally disintegrate into small turbulent eddies. The downstream wave development is a process which is controlled by the upstream conditions. By imposing a periodic surging of known frequency and amplitude at the jet exit, it is possible to follow the development of the spatial instability waves. Figure 3a shows the centerline distributions of the magnitude and phase of the longitudinal velocity fluctuations as the instability wave propagates downstream. Here the rms value of the velocity fluctuations is normalized with respect to the exit velocity U_e, and the axial distance x with respect to the diameter D. The forcing frequency is 348 Hz which corresponds to a St of 0.38 based on U_e and D. This value of St is close to the value of 0.37 reported by Crow and Champagne[3] for maximum growth at 0.5% forcing. From the linearized theory of the shear layer stability an exponential growth of the instability wave is predicted.[5] Even though the initial amplitude of the instability wave is sufficiently small, such that a linear theory might be applicable, the experimental results indicate that the growth rate does not follow the stability analysis. This is probably due to the background turbulence or the change in meanflow velocity profiles for increasing x/D , which the linear theory does not account for. Also, as the wave grows in amplitude, nonlinear effects will become important, and higher harmonics are generated which will retard the growth of the fundamental. This can be seen from some of the output signals of the eductor showing the distortion of the waveform for increasing distance downstream of the nozzle.

The maximum amplitude occurs at a distance of approximately 4½ diam. which is near the tip of the potential core and from there on the amplitude decreases. The distance from the jet exit to this point of maximum growth is about 2½ wavelengths. This is close to Becker and Massaro's[2] results based on optical observation of the breakaway length, where the waves fold or roll back to form vortices. The phase angle is measured relative to the signal that drives the loudspeaker. Taking the phase at $x = 0$ as reference, the change in the phase angle $\Delta\phi$ is plotted with distance. The curve is quite linear and the average phase velocity can be calculated from these results.

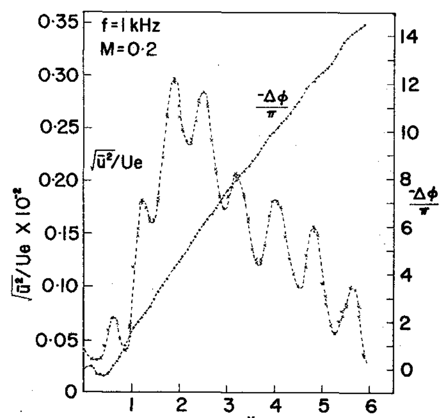

Fig. 4 Magnitude and phase distributions of instability wave along jet axis at $St = 1.1$ (fundamental mode).

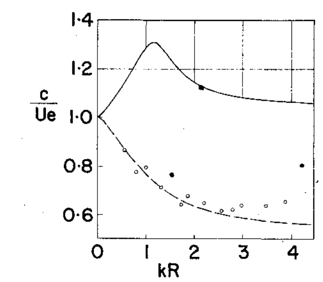

Fig. 5 Phase velocity vs wavenumber.

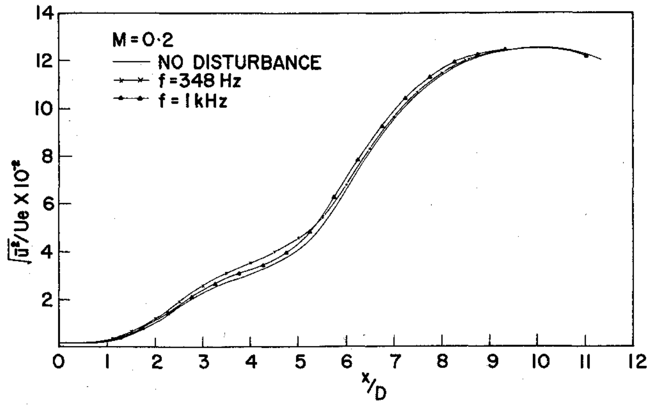

Fig. 6 *u* component of turbulent intensity along jet axis.

The magnitude and phase of the first harmonic can be obtained by doubling the frequency of the reference signal to the lock-in amplifier. In Fig. 3b the growth of the first harmonic with distance is shown. A number of maxima and minima in the amplitude are detected but the maximum growth still occurs near the tip of the potential core. Similar behavior of the amplitude, with distance from the nozzle, occurs at higher frequencies, as illustrated in Fig. 4. This shows the fundamental mode at a frequency of 1 kHz corresponding to a Strouhal number of 1.1. The amplitude peaks occur at fairly regular intervals of approximately $3/4D$, and the maximum amplitude is in the vicinity of $x/D = 2$, which is substantially further upstream of the tip of the potential core than that shown in Fig. 3a. The phase curve is fairly linear except near the jet exit. This indicates that the waves are progressing downstream, and the maxima and minima in the amplitude curve are not due to the presence of standing waves. An explanation of this phenomenon awaits more experimental studies which are presently in progress.

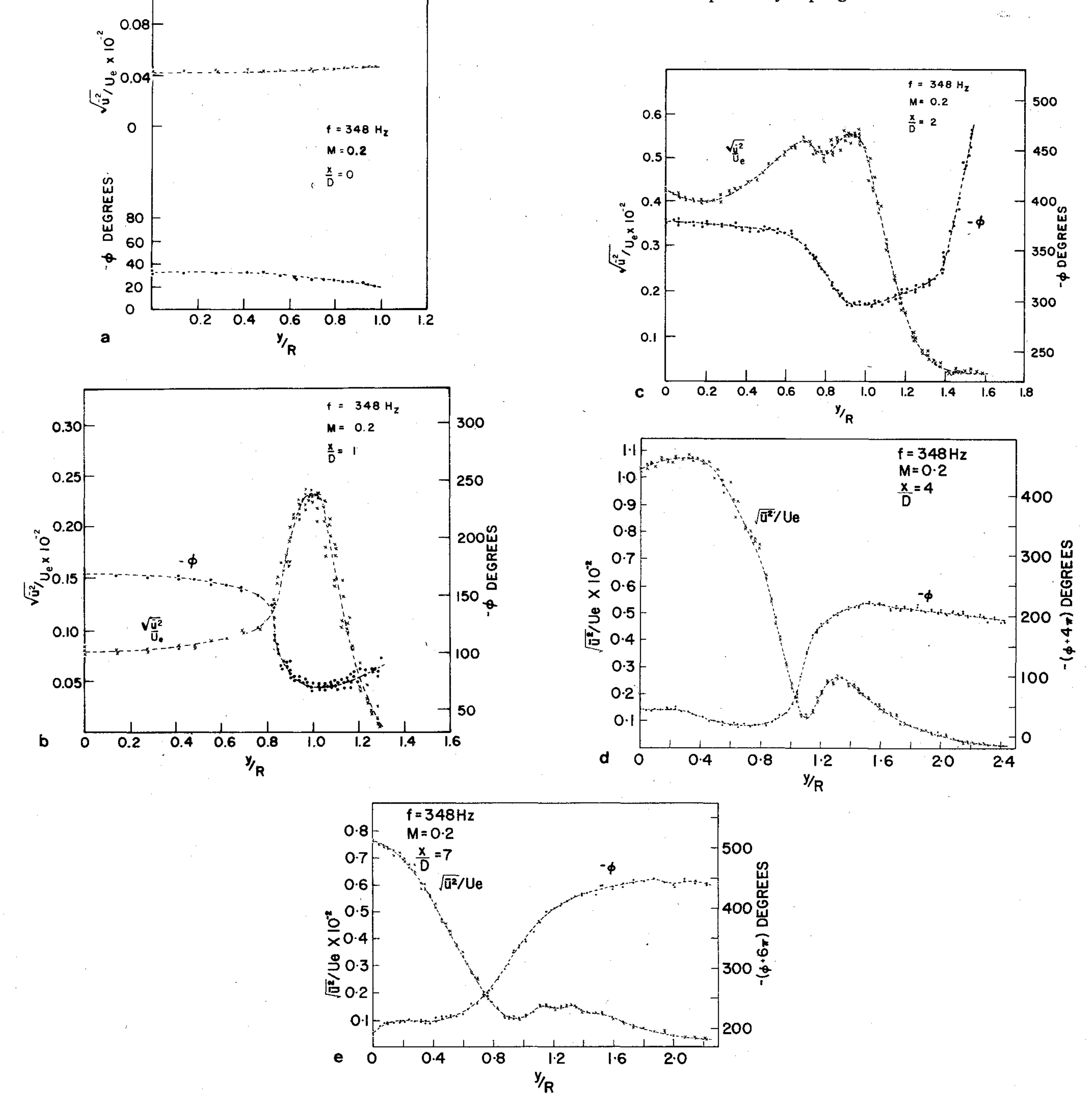

Fig. 7 Profiles of magnitude and phase of the fundamental mode across the jet at $ST = 0.38$, a) $x/D = 0$, b) $x/D = 1$, c) $x/D = 2$, d) $x/D = 0$, e) $x/D = 7$.

Knowing the variations of the phase angle ϕ with distance, the phase velocities can be determined. They are plotted in Fig. 5, which is obtained from Crow and Champagne.[3] Here the normalized phase velocity c/U_e is plotted against kR, where k is the wave number and R the radius of the nozzle. The two full circles are the results obtained from the phase measurements along the centerline of the jet and correspond to a frequency of 348 Hz and 1 kHz, respectively. The full square is the first harmonic corresponding to a fundamental frequency of 348 Hz. The open circles are the results from Crow and Champagne,[3] using a different technique in measuring the phase. The two curves correspond to the phase velocities predicted from linear stability theory for an incompressible jet having a top hat profile. The solid curve is for the spatial case and the dashed curve for the temporal case. Though the result for the 348 Hz and some of Crow and Champagne's[3] measurements fall closely to the temporal case, it cannot be said that temporal instability is more appropriate for a jet flow. Calculations of the phase velocity vs wave number show that when compressibility and finite shear layer thickness are included, the results are quite different from those obtained using the infinitely thin shear layer model. It is perhaps interesting to note that the experimental phase velocities for these two Strouhal numbers are approximately constant within the region of measurements even through the mean flow velocitiy profiles vary significantly.

Figure 6 shows the turbulent intensities along the jet centerline with and without surging at the exit plane. For weak forcing, the results show that the changes in intensities are not as significant as those of Crow and Champagne,[3] with a forcing of 2%. The potential core is more affected at a Strouhal number of 0.38, but in mixing region, it appears that the reverse is true at the higher frequency.

Figures 7a -7e show the magnitude and phase of the axial component of the velocity fluctuations across the jet for different values of x/D at a Strouhal number of 0.38. The radial distance from the jet axis y is normalized with respect to the radius R. Figure 7a is taken at $x/D=0$ just inside the nozzle. At a relatively low frequency or St the forcing is fairly uniform, and the amplitude and phase are quite constant across the nozzle.

At 1 diam away (Fig. 7b) the amplitude increases by about 50%. Inside the potential core the changes are quite gradual but on approaching the shear layer, the amplitude increases very rapidly, and reaches a peak at $y/R=1$ before decreasing. The results for y/R greater than 1.2 are not reliable because the mean flow is practically zero. The hot-wire results become meaningless if the mean velocity is not large enough to sweep away the hot-wire's wake. It is interesting to note that for y/R between 0.8 and 1.2 the curve looks somewhat like the turbulent intensity curves for the unforcet jet, and suggests that the background turbulence has a large influence on the waveform of the instability wave. The phase angle shows a similar behavior and there is a phase shift of nearly 90 deg in the shear layer.

At 2 diam away (Fig. 7c) the amplitude peak at $y/R=1$ can still be seen, and a second peak seems to have developed. Again the phase changes rapidly from the potential core to the shear layer and there is another sharp phase shift of opposite sign toward the jet boundary.

At 4 diam away (Fig. 7d), which is near the tip of the potential core, the phase angle curve shows two regions of fairly uniform phase. In between, the phase changes by nearly 180 deg. From Becker and Massaro's[2] results the vortex sheet will roll up to form vortices at about 2½ wavelengths from the jet exit. This corresponds closely to the tip of the potential core where the amplitude of the instability wave has grown to its maximum. The existence of vortices provides an explanation for the 180 deg phase reversal. The amplitude curve looks quite different from those of Figs. 7b and 7c. The peak moves near to the axis and in Fig. 7e the maximum occurs on the jet axis while the phase change across the jet is more gradual. At this value of x, which is 7 diam away from the nozzle exit, the mean flow velocity profile has thickened substantially but a phase shift of approximately 180 deg is still present.

Conclusions

By means of a weak periodic forcing at the jet exit, the development of spatial instability waves in a round subsonic jet has been studied experimentally from hot-wire measurements. The waveform of the instability wave across the jet depends strongly on the mean flow velocity profile, and the 180 deg phase reversal detected downstream of the region of maximum amplitude indicates the formation of vortices. The growth rate of the instability waves does not follow an exponential form as predicted from linear theory, and at higher frequencies the centerline variations of the wave amplitude show a number of maxima and minima. The phase angle increases fairly linearly along the axis, and from these measurements the average phase velocity can be obtained. For weak forcing, a preferred mode with the highest amplitude growth is found at a Strouhal number of about 0.38.

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